Math 115A A, Lecture 2
Real Analysis

Sample Midterm 2

Instructions: You have 50 minutes to complete the exam. There are five problems, worth a total of fifty points. You may not use any books or notes. Partial credit will be given for progress toward correct proofs.

Write your solutions in the space below the questions. If you need more space use the back of the page. Do not forget to write your name in the space below.

Name: ______________________________________

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

Problem 1.

(a) [5pts.] State the dimension theorem. Be sure to include all relevant hypotheses.

(b) [5pts.] Starting from the dimension theorem, give an argument that if $T : V \to W$ is a linear map between two n-dimensional vector spaces, then the T is onto if and only if T is one-to-one.

Problem 2.

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the projection onto the x-axis along the line $y = 2x$.

(a) [5pts.] Give a basis of eigenvectors of T, with corresponding eigenvalues.

(b) [5pts.] Find the matrix of T in the standard basis for $\mathbb{R}^2$.

Problem 3.

Let $T : V \to V$ be a linear transformation from a finite-dimensional vector space to itself.

(a) [5pts.] Prove that $T^2 = T_0$ if and only if $R(T) \subset N(T)$.

(b) [5pts.] Let β be a basis for V. If $T^k = T_0$ for some k, what is $\det([T]_\beta)$?

Problem 4.

(a) [5pts.] What does it mean for two vector spaces V and W to be isomorphic?

(b) [5pts.] Let V and W be finite dimensional vector spaces. Prove that a linear transformation $T : V \to W$ is an isomorphism if and only if it maps any basis β for V to a basis $T(\beta)$ for W.

Problem 5.

(a) [5pts.] Let $A = \lambda I_n$ be a diagonal $n \times n$ matrix all of whose diagonal entries are λ. Prove that if B is an $n \times n$ matrix similar to A, then $B = A$.

(b) [5pts.] Prove that the transformation

$$\begin{aligned} T : \mathbb{R}^2 &\to \mathbb{R}^2 \\ (x, y) &\mapsto (x + y, y) \end{aligned}$$

is not diagonalizable.